CALCULATION OF QUASI-STATIC CHARACTERISTICS OF MICROSTRIP ON ANISOTROPIC SUBSTRATE USING MAPPING METHOD

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In this paper a mapping transforms a original microstrip line on anisotropic substrate into another on isotropic dielectric. This method is applied to compute the quasi-static impedance of single microstrip on an anisotropic sapphire substrate.

Introduction

Recently various articles have appeared concerning the microstrip line on anisotropic substrate. Owens et. al. [1] have studied the impedance of a microstrip structure similar to Fig. 1(a) in which the substrate is an anisotropic sapphire crystal of relative permitivity tensor strictly diagonal. These authors employed the method of finite differences and obtained an equivalent isotropic relative permitivity which permits the computations of the characteristic impedance of this structure in TEM mode. N.B.Alexopoulos and C.M. Krowne [2] have studied covered single and couples microstrips on anisotropic substrate. These authors obtained a solution by solving Laplace's equations in the dielectric regions subject to the proper boundary conditions and proceeded to compute Green's functions with programs based on the moments approach. In this way, the results obtained for the single microstrip case differ from Owens in 5%.

In this paper a mapping from R_z to R_w transforms a original microstrip line on anisotropic substrate into another on isotropic dielectric. In this mapping the line capacitance is invariant. Therefore, the mode TEM microstrip impedance is calculated by studying the corresponding equivalent structure utilizing any of the known methods of microstrip on isotropic substrate analysis. The method is particular applied to the line of Fig. 1(a). The Wheeler method has been used to calculate the impedance of the equivalent line.

Analysis

The method that is used to study the characteristics of microstrip lines on anisotropic substrate is based on a mapping proposed by S.Kusase and Terakado [3] concerning bidimensional electrostatic problems. These authors consider the mapping between Z and W complex plane. If R_z and R_w are two regions in $z = x + iy$ plane and $w = u + iv$ plane respectively, the mapping function is expressed as:

$$u = u(x,y) \qquad v = v(x,y) \tag{1}$$

For this mapping the potencial of the point (x,y) in R_z is equal to the potencial of the point (u,v) in R_w. Also, the electrostatic energy of the system is invariant.

Lastly, if the mapping function (1) are selected by which the matrix:

$$[M] = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial v}{\partial x} \\ \frac{\partial u}{\partial y} & \frac{\partial v}{\partial y} \end{bmatrix} \tag{2}$$

verifies the relation:

$$[M][M]^t/\det[M] = \sqrt{\det[\varepsilon_z]}\,[\varepsilon_z]^{-1} \tag{3}$$

the permitivity tensor $[\varepsilon_z]$ of the dielectric in the R_z region is transformed into:

$$[\varepsilon_w] = \varepsilon_w [I]$$

whereby:

$$\varepsilon_w = \sqrt{\det[\varepsilon_z]} \tag{4}$$

In this mapping the coefficients of capacitance of a system of n conductors are invariant. Henceforth, this method is utilizable in the analysis of microstrip characteristics.

In Fig. 2(a) the microstrip line with n strips is shown. The substrate is an anisotropic dielectric of permitivity tensor:

$$[\varepsilon_z] = \varepsilon_o \begin{bmatrix} \varepsilon^*_{xx} & \varepsilon^*_{xy} & 0 \\ \varepsilon^*_{xy} & \varepsilon^*_{yy} & 0 \\ 0 & 0 & \varepsilon^*_{zz} \end{bmatrix} \tag{5}$$

The mapping function that is proposed for the study of quasi-static characteristics of this structure is:

$$u = ax + by \qquad v = dx + cy \tag{6}$$

In this case:

$$[M] = \begin{bmatrix} a & d \\ b & c \end{bmatrix} \tag{7}$$

In agreement with (4) the equivalent structure in R_w has an isotropic dielectric substrate of permitivity:

$$\varepsilon_w = \varepsilon_o \sqrt{\varepsilon^*_{xx}\varepsilon^*_{yy} - \varepsilon^{*2}_{xy}} \tag{8}$$

The equation (3) must be considered now. The following equations system is obtained:

$$a^2 + d^2 = \frac{\Delta}{\varepsilon_w} \varepsilon_o \varepsilon^*_{yy}$$

$$ab + dc = -\frac{\Delta}{\varepsilon_w} \varepsilon_o \varepsilon^*_{xy} \qquad (9)$$

$$b^2 + c^2 = \frac{\Delta}{\varepsilon_w} \varepsilon_o \varepsilon^*_{xx}$$

where:

$$\Delta = ac - db$$

In this equations system there is only two independent equations, one is therefore free to take two parameters:

$$a = 1 \qquad c = \frac{\varepsilon_w}{\varepsilon_o \varepsilon^*_{yy}}$$

$$d = 0 \qquad b = -\frac{\varepsilon^*_{xy}}{\varepsilon^*_{yy}} \qquad (10)$$

and the mapping equations:

$$u = x + by$$

$$v = cy$$

Lastly, the equivalence line (Fig. 2(b)) in R_w is another open microstrip of the same geometric arrangement, where the substrate is an isotropic dielectric of relative permitivity:

$$\varepsilon'_r = \frac{\varepsilon_w}{\varepsilon_o} = \sqrt{\varepsilon^*_{xx}\varepsilon^*_{yy} - \varepsilon^{*2}_{xy}} \qquad (11)$$

and thickness:

$$h' = h\sqrt{\frac{\varepsilon^*_{xx}}{\varepsilon^*_{yy}} - \frac{\varepsilon^{*2}_{xy}}{\varepsilon^{*2}_{yy}}} \qquad (12)$$

The strips width:

$$w'_i = w_i \qquad (13)$$

and:

$$d'_i = d_i$$

Results

The proposed method has been applied to the computation of the characteristic impedance of single microstrip on an anisotropic sapphire substrate (Fig. 1(a)) whose tensor permitivity is:

$$[\varepsilon_z] = \varepsilon_o \begin{bmatrix} \varepsilon^*_{xx} & 0 & 0 \\ 0 & \varepsilon^*_{yy} & 0 \\ 0 & 0 & \varepsilon^*_{zz} \end{bmatrix} \qquad (14)$$

$\varepsilon^*_{xx} = \varepsilon^*_{zz} = 9.40$, $\varepsilon^*_{yy} = 11.60$ [1] .

The equivalent line substrate (Fig. 1(b)) has a relative permitivity from (11):

$$\varepsilon'_r = \sqrt{\varepsilon^*_{xx}\varepsilon^*_{yy}} \qquad (15)$$

and a thickness from (12):

$$h' = h\sqrt{\frac{\varepsilon^*_{xx}}{\varepsilon^*_{yy}}} \qquad (16)$$

Finally from (13):

$$w' = w \qquad (17)$$

Characteristic impedance of microstrip on sapphire substrate Z_o, and equivalent line Z'_o, can be obtained as follows:

$$Z_o = \sqrt{\frac{120\pi\,\varepsilon_o Z_v}{C}}$$

and: (18)

$$Z'_o = \sqrt{\frac{120\pi\,\varepsilon_o Z'_v}{C}}$$

Z_v and Z'_v are, respectively, characteristic impedance of those microstrips if the dielectric sheet is removed. Therefore, Z_o can be obtained as follows:

$$Z_o = Z'_o\sqrt{\frac{Z_v}{Z'_v}} \qquad (19)$$

Succesively, the impedance is computed through the equivalent microstrip on an isotropic substrate, by the Wheeler method. For a simple microstrip (Fig. 1(b)) Wheeler [4] obtains the following equation to compute the microstrip line impedance:

$$Z'_o = \frac{42.4}{\sqrt{\varepsilon'_r+1}} \ln\left\{1 + \frac{4h'}{w'}\left[\frac{14+8/\varepsilon'_r}{11}\cdot\frac{4h'}{w'} + \sqrt{\left(\frac{14+8/\varepsilon'_r}{11}\right)^2\cdot\left(\frac{4h'}{w'}\right)^2 + \frac{1+1/\varepsilon'_r}{2}\pi^2}\right]\right\} \qquad (20)$$

with a marginal error between 1 and 2%. The advantage of this method is the enormous simplicity to compute.

Tabulation of impedance of microstrip on anisotropic substrate is inmediate with a digital pocket calculator.

Graph of Fig. 3 shows the impedance of microstrip line on anisotropic sapphire substrate versus w/h. The error is less than 3% for w/h = 10 and becomes less than 1% for w/h = 0.1, bettering the results of Alexopoulos and Krowne for the same structure, with the advantage of easier and faster computation.

Conclusions

The quasi-static characteristics of a microstrip on anisotropic substrate has been analyzed. A mapping proposed by Kusase and Terakado for electrostatic problems has been used. The microstrip line configuration is mapped into another on isotropic substrate. This method has been applied to the particular case of a single microstrip. The results obtained from the method that has been described in this paper are in agreement with other authors. On the other hand, there is an important diminution of computation time. Henceforth, graphs of the impedance of microstrip lines on anisotropic substrate versus w/h are obtained easily.

References

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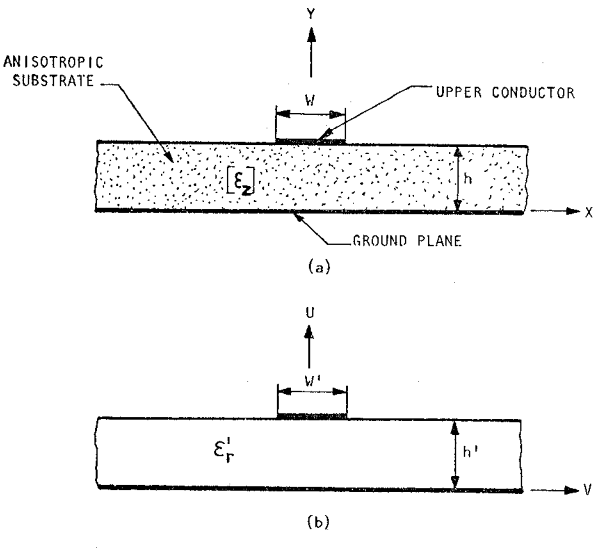

Fig. 1. Cross section of (a) microstrip on anisotropic substrate and (b) equivalent microstrip on isotropic substrate.

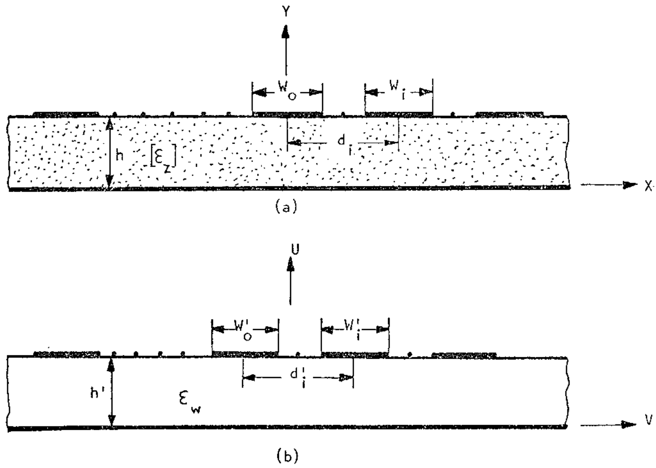

Fig. 2. Cross section of (a) multistrip on anisotropic substrate, (b) equivalent multistrip on isotropic substrate.

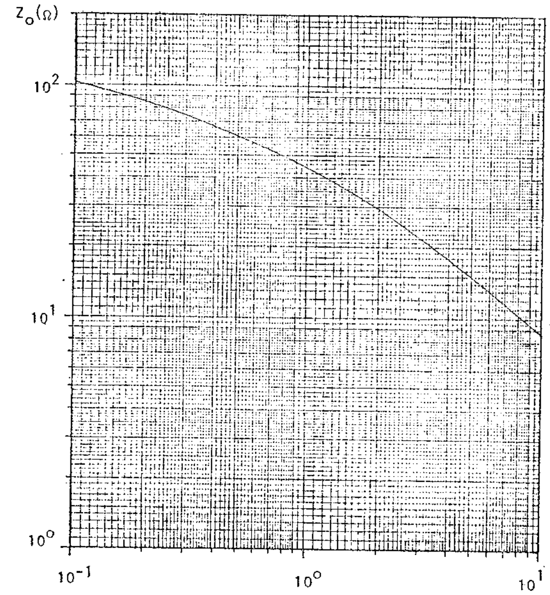

Fig. 3. Impedance of microstrip on an anisotropic sapphire as function of w/h.